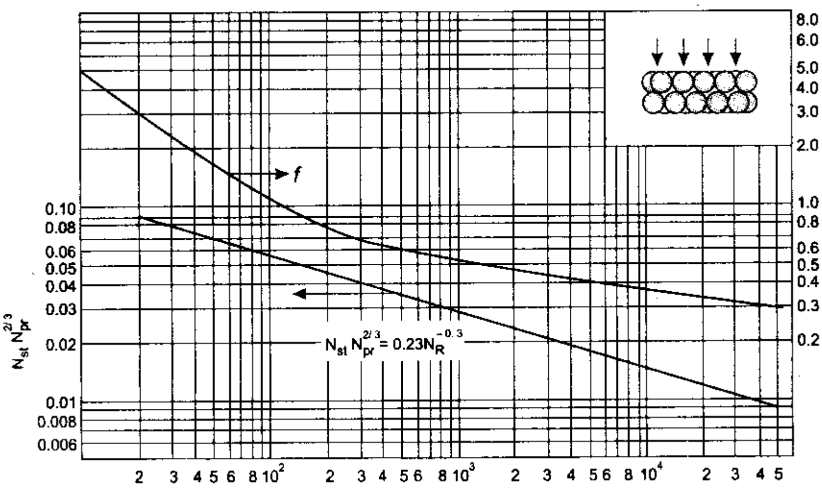

FIGURE A12.6 Data for an infinite, randomly stacked sphere matrix with porosity varying from 0.37 to 0.39

TABLE A12.1 Heat transfer and friction data for sphere bed matrices (Kays and London) (Random packing, p = 0.37 to 0.39)

Reynolds number, N_R	$N_{St} N_{Pr}^{2/3}$	f
50000	0.0089	0.30
20000	0.0118	0.34
10000	0.0144	0.37
5000	0.0178	0.41
2000	0.023	0.47
10000	0.029	0.52
500	0.0355	0.59
200	0.046	0.80
100	0.056	1.10
50	0.069	1.65
20	0.091	3.0
10	0.112	5.2

i.e.

$h_c(T - T_s)(A_h/L)\,dx = m_s\,(dx/L)\,c_s\,(\partial T_s/\partial \tau)$ ...(2)

Solving Eqs. 1 and 2, we get the partial differential equation for the temperature of gas flowing through the regenerator:

$$[\partial^2 T/(\partial x.\partial \tau)] + [h_c A_h/(m\,c_p\,L)]\,(\partial T/\partial \tau) + [h_c\,A_h/(m_s c_s)]\,(\partial T/\partial x) = 0 \quad ...(3)$$

From Eq. (3), we observe that two important dimensionless quantities are involved in the analysis of a regenerator, i.e.

$N_{tu} = h_c\,A_h/(m\,c_p)$ = Number of heat transfer units, and

$F_n = h_c\,A_h/(m_s\,c_s f)$ = Frequency number, where $f = 1/P$ = frequency of switching the hot and cold stream, P = heating or cooling period.

$f.\tau$ = dimensionless time.

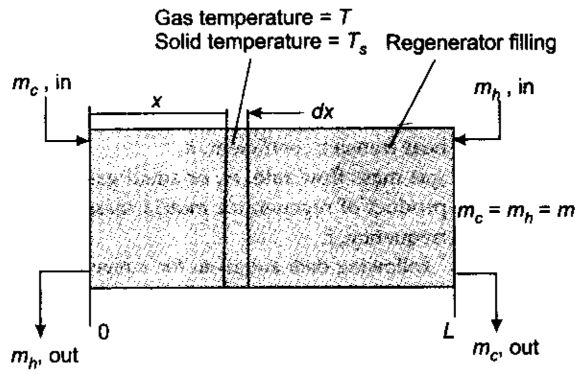

FIGURE A12.7 NTU analysis of a regenerator

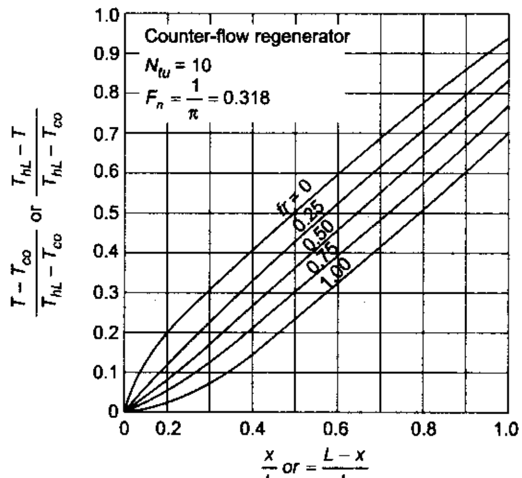

Fig. A12.8 Gas temperature distribution for a counter-flow regenerator as a function of time. For the regenerator, $N_{tu} = \frac{h_c \cdot A_h}{m \cdot c_p} = 10$ and $F_n = \frac{h_c \cdot A_h}{m_s \cdot c_s \cdot f} = 1/\pi$. During the heating period (when the cold gas is flowing through the regenerator, use $(T - T_{co})/(T_{hL} - T_{co})$ and x/L as coordinates; for the cooling period (when the warm gas is flowing through the regenerator), use $(T_{hL} - T)/(T_{hL} - T_{co})$ and $(L - x)/L$ as coordinates. T_{hL} is entering temp. of warm stream (at $x = L$) and T_{co} is the entering temp. of cold gas stream (at $x = 0$).

FIGURE A12.8 Gas temperature distribution in a counter-flow regenerator

Eq. 3 has been solved numerically for temperature distribution of gas, by Hausen for steady state cyclic operation of a regenerator. Results for a particular case of $N_{tu} = 10$ are shown in Fig. A 12.8, as an example.

Once the temperature distribution is known, actual energy transferred is calculated as:

$$Q_{\text{actual}} = \int_0^{\frac{1}{f}} \int_0^L h_c \cdot \frac{A_h}{L} \cdot (T - T_s)\, dx\, d\tau \qquad \text{...(4)}$$

Maximum possible heat transfer in the regenerator occurs when the gas is heated from T_{co} (i.e. temperature of cold gas entering at $x = 0$) to the temperature T_{hL} (i.e. temperature of hot gas entering the regenerator at $x = L$). We get:

$$Q_{\text{ideal}} = \frac{m \cdot c_p \cdot (T_{hL} - T_{co})}{f} \qquad \text{...(5)}$$

Then, regenerator effectiveness ε, is given by:

$$\varepsilon = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}}$$

i.e.

$$\varepsilon = \frac{h_c \cdot A_h}{m \cdot c_p} \cdot \int_0^1 \int_0^1 \left(\frac{T - T_s}{T_{hL} - T_{co}} \right) d\left(\frac{x}{L} \right) d(f \cdot \tau) \qquad \text{...(6)}$$

Hausen's numerical solution for the effectiveness of a regenerator as a function of frequency number and number of transfer units, is shown in Fig. A 12.9.

It is clear from Fig. A12.9 that for a large effectiveness, we need a small frequency number (F_n) and a large number of transfer units (N_{tu}), *i.e.* for a large effectiveness of a regenerator, the requirements are:

(i) large heat transfer coefficient, h_c

(ii) small gas mass flow rate, m, or small gas capacity rate $m.c_p$

(iii) large product of regenerator matrix mass and its specific heat, $m_s.c_s$

(iv) large frequency, f.

Example A12.2. Following data are given for a regenerator:

heat transfer coefficient = 640 W/(m²K), heat transfer area per unit length = 100 m²/m, mass of matrix solid per unit length = 8 kg/m, specific heat of matrix material = 800 J/(kgK), frequency of operation = 60 cpm (= 1 cycle/s), mass flow rate of gas through regenerator = 0.013 kg/s and specific heat of gas = 5200 J/(kgK). Desired effectiveness of this regenerator, operating in a counter-flow mode, is 0.95. Determine the length of the regenerator required.

Solution.

Data:

$h_c := 640\ \text{W/(m}^2\text{K)} \qquad \frac{A_h}{L} = 100\ \text{m}^2\text{/m} \qquad \frac{m_s}{L} = 8\ \text{kg/m} \qquad c_s := 800\ \text{J/(kgK)} \qquad f := 1\ \text{cycle/s} \qquad m := 0.02\ \text{kg/s}$

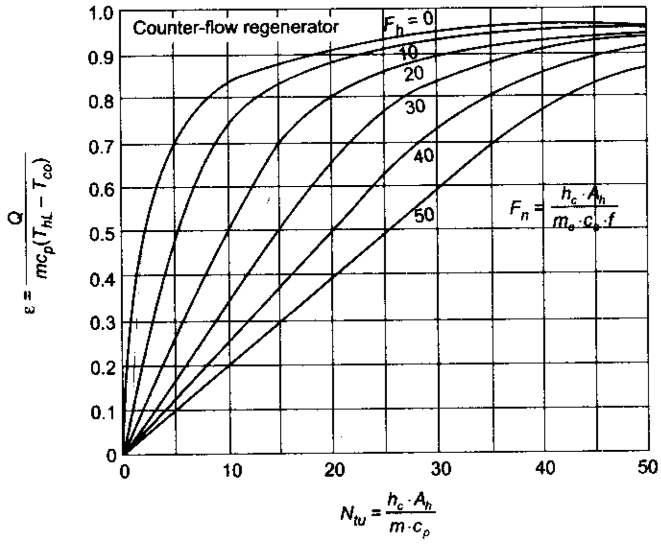

FIGURE A12.9 Effectiveness of a counter-flow regenerator

$c_p := 5200$ J/(kgK) $\quad \varepsilon := 0.95$

Frequency number:

We have:
$$F_n = \frac{h_c \cdot \left(\frac{A_h}{L}\right)}{\left(\frac{m_s}{L}\right) \cdot c_s \cdot f}$$

i.e.
$$F_n = \frac{640 \cdot 100}{8 \cdot 800 \cdot 1}$$

i.e.
$$F_n = 10 \qquad \textit{(frequency number)}$$

Number of heat transfer units:

For a $F_n = 10$ and $\varepsilon = 0.95$, get the value of N_{tu} from Fig. A 12.9:

We get: $N_{tu} = 45$

But,
$$N_{tu} = \frac{h_c \cdot A_h}{m \cdot c_p}$$

Therefore, heat transfer area required:
$$A_h := \frac{N_{tu} \cdot m \cdot c_p}{h_c}$$

i.e.
$$A_h = 7.313 \text{ m}^2 \qquad \textit{(heat transfer area required)}$$

Therefore, regenerator length required:
$$L = \frac{A_h}{\frac{A_h}{L}}$$

i.e.
$$L = \frac{7.313}{100}$$

i.e.
$$L = 0.073 \text{ m} = 7.3 \text{ cm} \qquad \textit{\textbf{(length of regenerator required.)}}$$

Regenerator ineffectiveness (1 – ε) vs. NTU graphs for cryogenic regenerators:

As stated in the text, cryogenic regenerators, generally, have large values of modified NTU (i.e. NTU_{mod}), of the order of 100 or more. It may be observed that in the usual ε–NTU graphs, the value of ε approaches unity asymptotically; so, for cryogenic heat exchangers, it is more instructive and convenient to draw regenerator inef-

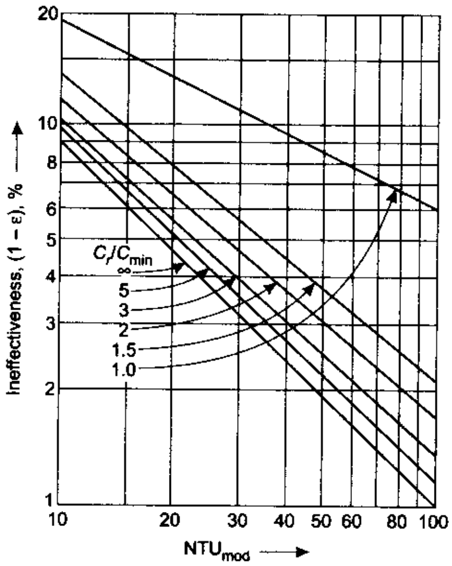

FIGURE A12.10 Regenerator ineffectiveness as a function of N_{tu0} and matrix capacity rate ratio (C_{min}/C_{max} = 1)

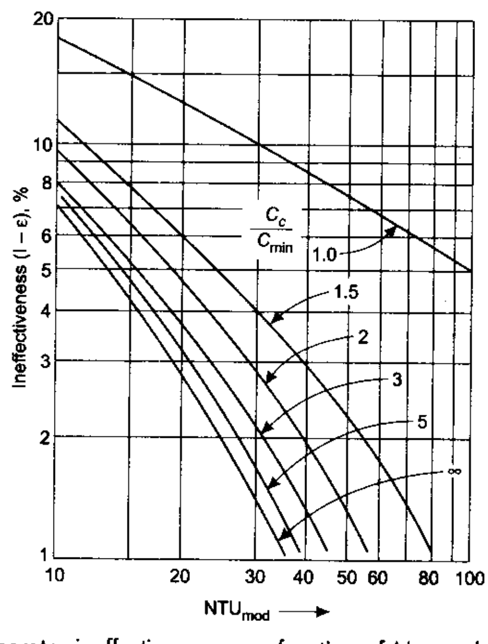

FIGURE A12.11 Regenerator ineffectiveness as a function of N_{tu0} and matrix capacity rate ratio (C_{min}/C_{max} = 0.95)

fectiveness $(1 - \varepsilon)$ against NTU_{mod} in log–log coordinates. Two sample graphs, one for $C_{min}/C_{max} = 1$, and the other for $C_{min}/C_{max} = 0.95$, are shown in Fig. A12.10 and Fig. A12.11, respectively. (Ref: Compact Heat Exchangers by Kays and London).